$$\sum_{e_{in,j}} e_r(e_{in}, j, SS) = \sum_{e_{out,j}} e_p(e_{out}, j, SS) + accumulation$$
$$\forall j \in SS \quad \forall SS \in S \quad \forall e_{in,j} \in E_{in,j} \quad \forall e_{out,j} \in E_{out,j} \quad e = E_{in} \cup E_{out}$$
$$j \in \{HE,\ fermenter, MX, SP, VF, ...\}$$
$$SS \in \{A, B, C, D, E, F, G, U\} \tag{2}$$

All input heat flows e_r are equal to all output heat flows e_p and possible heat accumulation. All those heat flows are determined with heat streams (e_{in} and e_{out}), processing units j and subsystems SS. Accumulation of mass and energy is zero in the most cases of balances. Model needs to make a correlation between input materials and final products. That means, quality and quantity of molasses that is entering into the system will cause all changes in quantity and quality of produced products and heat energy demand

Balances are made for every processing unit j in every subsystem SS, parts of the whole process system S. Energy balances of inlet and outlet energy streams are determined by additional heat *Q*, work of fluid *W*, enthalpy of fluid *h*, its kinetic $V^2/2$ and potential energy $g \cdot z$ (equations 3-5). Potential and kinetic energies are taken to be constant and additional heat has values only in cases with biochemical transformation of sugars.

$$E_{in,j} = Q_{in,j} + W_{in,j} + \sum_{m_{in,j}} m_{in,j}\left(h_{in,j} + \frac{V_{in,j}^{\;2}}{2} + g \cdot z_{in,j}\right) \cdot \tau \tag{3}$$

$$E_{out,j} = Q_{out,j} + W_{out,j} + \sum_{m_{out,j}} m_{out,j}\left(h_{out,j} + \frac{V_{out,j}^{\;2}}{2} + g \cdot z_{out,j}\right) \cdot \tau$$
$$j \in SS, \quad \forall m_{in,j,S} \in M_{in,j,S}, \quad \forall m_{out,j,S} \in M_{out,j,S}, m = M_{in} \cup M_{out}, \quad \forall SS \in S$$
$$z_{in,j,S} = z_{out,j,S}, \quad V_{in,j,S} = V_{out,j,S}, \quad W_{in,j,S} = W_{out,j,S}$$
$$j \in \{HE,\ fermenter, MX, SP, VF, ...\} \quad SS \in \{A, B, C, D, E, F, G, U\} \tag{4}$$

Equations 3 and 4 represents inlet and outlet energies for certain process unit situated in certain subsystem SS. Every inlet or outlet mass flow m is part of all inlet or outlet mass flows M streams for determined processing unit j. Every subsystem SS is part of the whole production system S.

The next step is using some software to connect all mass and energy balances of all processing units between themselves. MS Excel is software with that kind possibilities. Process scheme as background of excel worksheet is used to represent all processing units and to implement their mass and heat balances. Every exit stream from processing unit is determined from the process in unit and all inlet streams. All inlet streams are exiting from units with processes earlier than the process in certain processing unit. According to that implementation of balances into processing scheme, mathematical model is created for whole system and prepared for simulation of production. Different values of parameters those determine inlet streams give different values of parameters for all streams that exit the system. So, this model could be used for simulation and optimization of the presented production system.

The aim of this work is to find optimal values of process parameters for created mathematical model in relation with the quantity of heat consumption. So, the sum of all heat transferred through heat exchangers from hot utility to fluid or from fluid to cold utility need to be minimized. In this case, the quantity of produced products needed to be maximized in the same time (equation 8).

Fermenter mass and heat balances are in relation to the behavior of yeast cells. Growth of yeast cells has exponential function related with time and yeast cell`s specific coefficient of growth μ. Therefore, all heat and mass values are changing exponentially according to biochemical reaction r (equations 5 and 6). As a part of heat minimization, feeding of molasses in fermentation is optimized and causes different fermentation time and changes in quantity of produced yeast biomass. That means lesser quantity of molasses will use lesser quantities of cold utility to regulate fermentation temperature, but that will cause lower quantity of produced yeast biomass.

$$m_{in,\,fermenter} = m_{out,\,fermenter} + r = m_{out,\,fermenter} + \frac{dm_{in,comp}}{d\tau} \tag{5}$$

$$r = \frac{dm_{in,comp}}{d\tau} = \pm m_{in,comp} \cdot e^{\mu\tau} \tag{6}$$

Simulation model is simplified for easier calculation. Mechanical work for all balances is taken to be zero. Heating of fermentation mash is simplified and related only with biochemical transformation of sugar (equation 7). Equation 7 show energy balance for process unit j as fermenter that belong to system S, and it is a sum of all heats of fluids that enter in fermenter, changing of heat as a result of biotransformation as well as work of fluids (W=0). There are some additional energy sources in fermentations such as air blowing, mixing of fermentation mash with airlift system or similar, environment influence, etc.

$$\Delta E = \Delta Q_j + \Delta W_j + \sum_{m_j} m_j \Delta h_j \cdot \tau$$
$$j = fermenter, \quad j \in S, \;\; \Delta W = 0 \tag{7}$$

Process stream's parameters in the production may vary in the particular range, related to the flexibility of equipment capacity. Every production unit has its own minimum and maximum of production. That is used in the mathematical model to get limitations, in purpose to find optimal values (table 1). One production line has its minimal capacity as a value for the highest value of minimum capacity for all processing units connected to that line. Similar is with its maximum capacity. The largest capacity of the production line has the smallest value of maximal capacity for all connected processing units.

Table 1. Limits given to constrain of the mathematical model in MS Solver to minimize heat consumption

parameters	Low limit	High limit
Total raw molasses [kg]	0.0	Any number of raw molasses needed for batch cycle
Inlet raw molasses temperature [ºC]	8.0	15.0
%sugar in raw molasses [mass%]	40.0	50.0
%sugar in molasses solution [mass%]	20.0	23.0
Raw molasses flow [kg/s]	1.6	2.0
Alcoholic mash flow [kg/s]	1.2	1.6
Number of cycles	Integer	1.0
Fermentation molasses [kg]	20000.0	24000.0

Values of selected parameters (table 1) can have any number within the set limits. MS Solver has an ability to optimize only one parameter value from the model (Figure 3). That is why a new parameter is introduced. This parameter represents the entire demand for heat per whole quantity of produced products. The value of this new parameter includes quantity of products and heat demand into one value. It can be calculated by using equation 8. The sum of all heat transferred from utility to the fluids and *vice versa* in production processes is divided by the sum of all quantity of end products as mass. Separate optimization of the production model according to the total heat demand and produced quantity of all end products give different values compared with optimal solution as the minimal value of heat demand per product unit (table 2). The optimal solution gives duration time of processes, such as molasses preparation time 61.6 h, aerobic fermentation per fermenter 20.3 h and vacuum filtration for fresh yeast 5.6 h. Changing of any parameter or constrains will change duration of processes in the mathematical model of production. All production`s processes duration could be determined according optimal values of parameters. That means optimal solution based on minimal heat demand and maximal production capacity will give optimal process scheduling. Determined values are given on Table 3. Those durations of processes describe process time scheduling for optimal conditions.

$$\min\left(heat\ demand\right)=\min\left(\frac{\sum_{u} heat}{\sum_{p} products}\right)\quad\left[\frac{kW}{kg}\right]\quad\left[\begin{array}{l}p=\{\,fresh\ yeast, dry\ yeast, alcohol\}\\ u=\{hot\ utility\ heat, cold\ utility\ heat\}\end{array}\right] \tag{8}$$

Table 2. Optimal values for heat demand and quantity of products determined for created mathematical model with MS Solver in different objectives of optimization

Optimization objective →	Minimal heat demand	Maximal production capacity	Minimal heat per product unit
Total heat [kW]	20922.0	27783.0	26590.0
Molasses per fermentation batch [kg]	20000.0	24000.0	24000.0
Ethanol [kg]	7884.0	10512.0	7884.0
Technical alcohol [kg]	133.9	178.6	133.9
Fresh yeast [kg]	41786.0	56463.0	56463.0
Dry yeast [kg]	7107.0	11428.0	11427.7

4. CONCLUSION

The first aim of production engineers is minimizing of costs and maximizing of production quantity. Many researchers use different optimization methods or combination of them. In this work, optimization is done for a case study for production of yeast and ethyl alcohol. Production system is containing various modes of processes, such as continuous, semi-continuous, batch and feed-batch. This production system is producing four different products: fresh yeast, dry yeast, refined ethyl alcohol and technical or denatured alcohol. All those processes have their processing time and production costs related to quantity of production and quality of raw materials. However, the duration of these processes and the deployment of equipment may be different. That requires a varied demand for heat exchange with utilities. Therefore, optimum duration is determined for each of the processes and the time of engagement of the equipment with minimal heat exchange. Optimal values of the model can be modified in accordance with the change of the process stream`s parameters. Moreover, the values for some variables can be taken as constant and other can be taken as variables. That is the case of input variables, such as temperature of raw molasses. That temperature is constantly changing. However, in purpose of the optimization of process duration, that temperature value could be taken as constant for a certain case.

In this work, are used three different objectives for optimization. That is minimization of heat demand, maximization of product's quantity and minimization of the common parameter that contain heat and product capacity. Minimization of heat demand, gives values for all processing parameters to have small quantity of heat consumption. In that situation quantity of manufactured end products is proportionally decrease with minimization of heat demand.

In second case production capacity is maximized. There, also have proportional changing of heat demand with production capacity. In the third case by using common parameter that connects heat demand and quantity of produced products shows different values for heat consumption and production capacity. Actually, products that need more heat for its production are minimized, and products that no needs so high quantities of heat for their production are maximized. The third case helps to determine the best values for minimal heat demand and maximal production. Values of some parameters for different streams in the optimized state are given in Appendix B.

Optimal scheduling and duration of processes involved in this model is the base for further minimization of production costs. Minimization would be done by changing of hot and cold utility demand with Pinch technology.

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